Algebra 2

5-01 *n*th Roots and Rational Exponents

Root

- If $a^2 = b$, then a is a ________________ (2^{nd}) root of b.
- If $a^n = b$, then a is the ____________________ root of b.

Parts of a radical

________ $\sqrt[3]{64}$ ____________

____

Rational Exponents

$$b^{1/n} = \sqrt[n]{b}$$

$$b^{m/n} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$$

Evaluate

$36^{1/2}$

$\left(\frac{1}{8}\right)^{-\frac{1}{3}}$

$27^{\frac{4}{3}}$

Find roots with a calculator

- The $\sqrt{x}$ or $\sqrt{}$ key is for ______________ roots (either radicand then key or key then radicand depending on calculator)
- The $\sqrt[x]{y}$ or $\sqrt[y]{x}$ or $\sqrt[x]{}$ is for ________ root (index → key → radicand OR radicand → key → index)

Try it with $\sqrt[4]{100}$

Steps to solve an equation with an exponent

1. ___________ the exponent term
2. Take the ___________ of both sides where the index is the ________________
- If the index is _________, put ______
3. _________
4. ___________ your answers!!!

Solve. Round to two decimal places, if necessary.

$5x^3 = 320$

$(x + 3)^4 = 24$

235 #7, 9, 11, 13, 15, 17, 19, 21, 23, 27, 31, 35, 37, 39, 43, 47, 49, 51, 53, 55 = 20

Algebra 2

5-02 Properties of Rational Exponents and Radicals

Properties of Rational Exponents

- $x^m \cdot x^n = x^{m+n}$
- $(xy)^m = x^m y^m$
- $(x^m)^n = x^{mn}$
- $\frac{x^m}{x^n} = x^{m-n}$
- $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$
- $x^{-m} = \frac{1}{x^m}$

$6^{1/2} \cdot 6^{1/3}$

$(27^{1/3} \cdot 6^{1/4})^2$

$(4^3 \cdot w^3)^{-1/3}$

$\frac{t}{t^{\frac{3}{4}}}$

Using Properties of Radicals

Product Property → $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Quotient Property → $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt[3]{25} \cdot \sqrt[3]{5}$

$\frac{\sqrt[3]{32x}}{\sqrt[3]{4x}}$

$\frac{1}{\sqrt{7}-2}$

Adding and Subtracting Roots and Radicals

1. Simplify the ______________
2. ________________ like terms

$5(4^{3/4}) - 3(4^{3/4})$

$\sqrt[3]{81} - \sqrt[3]{3}$

$2\sqrt[4]{6x^5} + x\sqrt[4]{6x}$

Writing Radicals in Simplest Form

1. Remove any _____________ roots
2. Rationalize ____________________

$\sqrt[4]{64}$

$\sqrt[4]{\frac{7}{8}}$

$\sqrt[5]{\frac{x^5}{y^8}}$

$\frac{18rs^{\frac{2}{3}}}{6r^{\frac{1}{4}}t^{-3}}$

242 #1-49 every other odd, 51, 55, 59, 63, 67, 71, 73, 81, 85, 87, 93, 95 = 25

Algebra 2

5-02A Properties of Rational Exponents and Simplifying Radicals

Properties of Rational Exponents

- $x^m \cdot x^n = x^{m+n}$
- $(xy)^m = x^m y^m$
- $(x^m)^n = x^{mn}$
- $\frac{x^m}{x^n} = x^{m-n}$
- $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$
- $x^{-m} = \frac{1}{x^m}$

$6^{\frac{1}{2}} \cdot 6^{\frac{1}{3}}$

$\left(27^{\frac{1}{3}} \cdot 6^{\frac{1}{4}}\right)^2$

$(4^3 \cdot w^3)^{-\frac{1}{3}}$

$\frac{t}{t^{\frac{3}{4}}}$

Simplifying Radicals

Remove any ____________ roots

Rationalize ___________________

$\sqrt[4]{64}$

$\sqrt[3]{625x^5}$

$\sqrt[4]{\frac{7}{8}}$

$\sqrt[5]{\frac{x^5}{y^8}}$

$\frac{1}{\sqrt{7}-2}$

$\frac{2}{3+\sqrt{5}}$

242 #1, 3, 5, 7, 9, 19, 21, 23, 25, 27, 29, 45, 47, 49, 95 = 15

Algebra 2

5-02B Operations with Radicals

Using Properties of Radicals

Product Property → $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Quotient Property → $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt[3]{25} \cdot \sqrt[3]{5}$

$\frac{\sqrt[3]{32x}}{\sqrt[3]{4x}}$

Adding and Subtracting Roots and Radicals

1. Simplify the _____________
2. _______________ like terms

$5\left(4^{\frac{3}{4}}\right) - 3\left(4^{\frac{3}{4}}\right)$

$\sqrt[3]{81} - \sqrt[3]{3}$

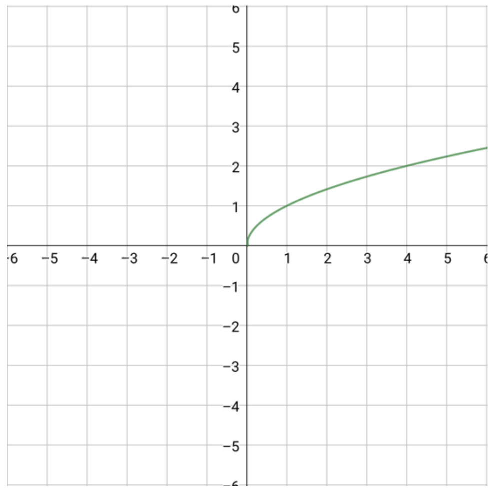
$2\sqrt[4]{6x^5} + x\sqrt[4]{6x}$

242 #11, 13, 15, 17, 35, 37, 39, 41, 43, 51, 63, 65, 71, 89, 91 = 15

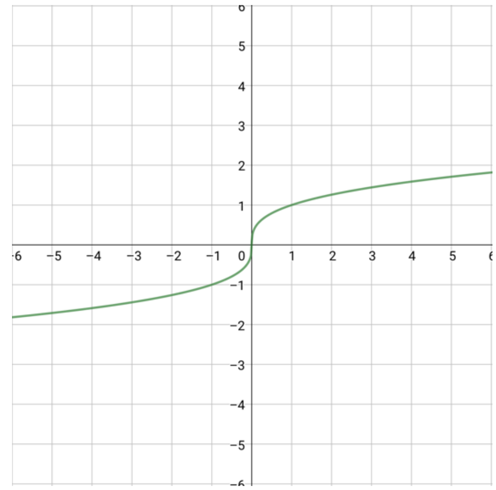
Algebra 2

5-03 Graphing Radical Equations

$y = \sqrt{x}$
Domain: ___________
Range: ____________



$y = \sqrt[3]{x}$
Domain: __________________
Range: ___________________



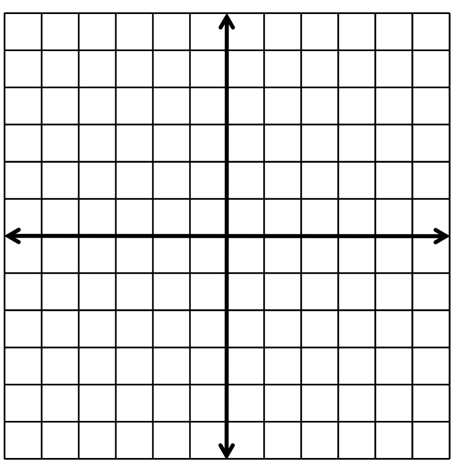
How graphs transform

$$y = a\sqrt{b(x-h)} + k$$
$$y = a\sqrt[3]{b(x-h)} + k$$

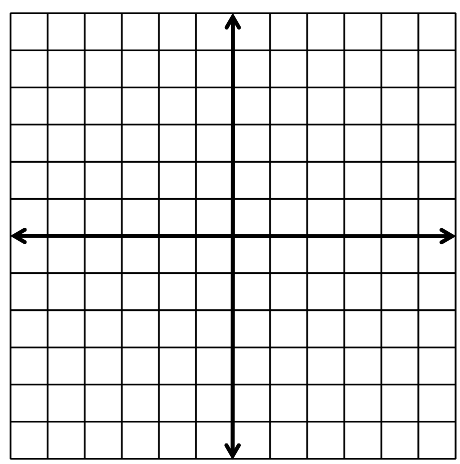
- Where
  - a ________________________ by factor of a
  - b ________________________ by factor of $\frac{1}{b}$
  - If a is −, _______________ over ________
  - If b is −, _______________ over ________
  - h ________________________
  - k ________________________
- Graph by making a ______________________.

Describe the transformation of f represented by g. Then graph each function.

$f(x) = \sqrt{x}$; $g(x) = \sqrt{x+2} - 3$



$f(x) = \sqrt[3]{x}$; $g(x) = -\sqrt[3]{2x}$



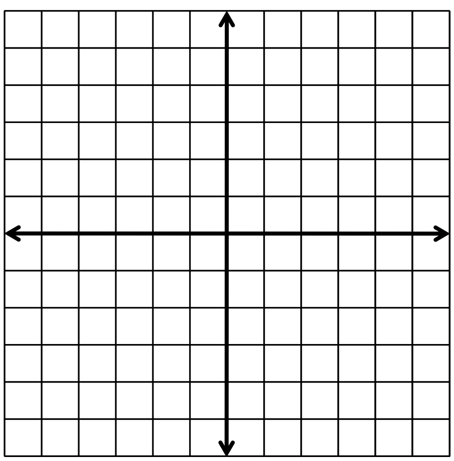
The function $E(d) = 0.25\sqrt{d}$ approximates the number of seconds it takes a dropped object to fall d feet on Earth. The function $J(d) = 0.63 \cdot E(d)$ approximates the number of seconds it takes a dropped object to fall d feet on Jupiter. How long does it take a dropped object to fall 81 feet on Jupiter?

Let the graph of g be a horizontal stretch by a factor of 3, followed by a translation 6 units right of the graph of $f(x) = \sqrt[3]{x}$. Write a rule for g.

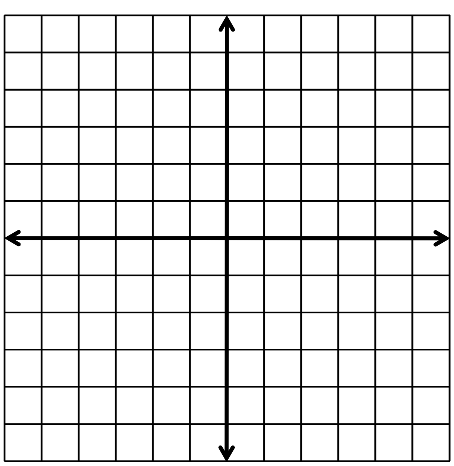
Graphing horizontal parabolas and circles

1. __________ the equation for y.
2. Create a ____________________.
3. __________ the points and __________ graph.

Graph $-\frac{1}{5}y^2 = x$. Identify the vertex and the direction that the parabola opens.



Graph $x^2 + y^2 = 49$. Identify the radius and the intercepts.



250 #1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 39, 41, 45, 49, 59, 67, 69, 71, 73, 79 = 20

Algebra 2

5-04 Solving Radical Equations and Inequalities

Radical Equation

Equation containing a ________________

Steps to Solve a Radical Equation

1. ________________ the radical
2. ________________ both sides to whatever the ________________ is (or the reciprocal of the exponent)
3. ________________
4. ________________ your answers!!!

$5 - \sqrt[4]{x} = 0$

$3x^{\frac{4}{3}} = 243$

$\sqrt{2x + 8} - 4 = 6$

$\sqrt{4x + 28} - 3\sqrt{2x} = 0$

$x + 2 = \sqrt{2x + 28}$

258 #1, 5, 9, 13, 17, 21, 25, 29, 31, 33, 37, 41, 45, 49, 51, 57, 61, 69, 77, 81 = 20

Algebra 2

5-05 Performing Function Operations

Ways to combine functions

- Addition: $(f+g)(x) = f(x) + g(x)$
- Subtraction: $(f-g)(x) = f(x) - g(x)$
- Multiplication: $(f \cdot g)(x) = f(x) \cdot g(x)$
- Division: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Given $f(x) = 5\sqrt{x}$ and $g(x) = -8\sqrt{x}$ find

$(f+g)(x)$

$(f-g)(x)$

$(f \cdot g)(x)$

$\left(\frac{f}{g}\right)(x)$

Let $f(x) = 2x^3 + 4x^2 - 8x + 4$ and $g(x) = 3x^3 - 5x^2 + 6x - 9$. Find $(f-g)(x)$ and state the domain. Then evaluate $(f-g)(-1)$.

Let $f(x) = x^3$ and $g(x) = \sqrt{x}$. Find $(fg)(x)$ and state the domain. Then evaluate $(fg)(4)$.

From 2010 to 2020, the populations (in thousands) of City M and City N can be modeled by $M(t) = 3.3t^3 + 12.1t^2 - 0.65t + 15.8$ and $N(t) = 2.5t^3 + 7.8t^2 + 0.41t + 11.9$, where t is the number of years since 2010. Find $(M - N)(t)$ and explain what it represents.

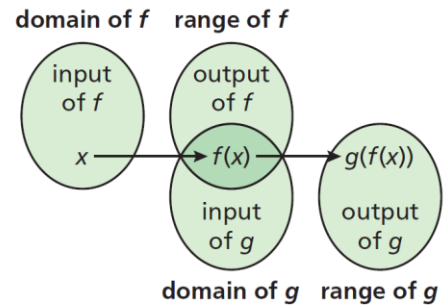
265 #1, 3, 5, 7, 9, 15, 17, 21, 23, 25, 27, 29, 35, 37, 39 = 15

Algebra 2

5-06 Composition of Functions

Composition

- Put one function __________ the other. (Like ___________________)
- Written __________
- Said "g of f of x"
- Means that the _____________ (range) of f is the __________ (domain) of g. Work from the inside out. Do $f(x)$ first then $g(x)$.
- $f(x)$ gets _______________________ into $g(x)$



Let $f(x) = \sqrt{3x-5}$ and $g(x) = x^2 + 1$. Find the indicated value.

a. $g(f(2))$

b. $f(g(3))$

c. $g(g(-3))$

Let $f(x) = 3x^{-1}$ and $g(x) = 4x - 5$. Perform the indicated operation and state the domain.

a. $f(g(x))$

b. $g(f(x))$

c. $f(f(x))$

The function $C(x) = 8.74x$ represents the cost (in dollars) of producing x shirts. The number of shirts produced in t hours is represented by $x(t) = 84t$. (a) Find $C(x(t))$. (b) Evaluate $C(x(40))$ and explain what it represents.

271 #1, 5, 9, 13, 17, 21, 25, 31, 33, 37, 43, 45, 47, 49, 51 = 15

Algebra 2

5-07 Inverse of a Function

Properties of Inverses

- x and y values are ____________________
- Graph is __________________ over the line ______________
- You can use the Horizontal Line test to determine if the __________________ of a function is also a function.
    - If a horizontal line can touch a graph __________________________, then the inverse is ________ a function.

Definition of inverses

- Two functions are inverses if and only if __________________ and __________________

Verify that $f(x) = 6 - 2x$ and $g(x) = \frac{6-x}{2}$ are inverses.

Finding inverses

- Inverses switch the x and y _______________________
1. _____________ x and y and ______________ for y.

Find the inverse

$y = 2x + 7$

$f(x) = x^4 + 2,\ x \leq 0$

The power (in watts) of a lightbulb that has a resistance of 240 ohms is represented by $f(x) = 240x^2$, where x is the electric current of a lightbulb in amperes. Find and interpret $f^{-1}(60)$.

279 #1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 43, 45, 47, 51, 57, 71, 77, 83, 87, 91 = 20

Algebra 2

5-Review

Take this test as you would take a test in class. When you are finished, check your work against the answers.

5-01

1. Evaluate $\sqrt[4]{150}$ using a calculator. Round the result to two decimal places if appropriate.
2. Evaluate $25^{\frac{3}{2}}$ using a calculator. Round the result to two decimal places if appropriate.
3. Solve $128 = 2(x-1)^6$

5-02

Simplify the expression. Assume all variables are positive.

4. $q^{\frac{7}{3}} \cdot q^{\frac{2}{3}}$
5. $\frac{x^{10}}{3x^6}$
6. $\sqrt[3]{81} + \sqrt[3]{24}$
7. $\sqrt[5]{64x^8y^{10}}$

5-03

Graph the function. Then state the domain and range.

8. $y = -2\sqrt[3]{x} + 1$
9. $y = \sqrt{x-2} - 3$
10. Describe the transformations to get $g(x) = 2\sqrt[3]{x+3}$ from $f(x) = \sqrt[3]{x}$.

5-04

Solve the equation.

11. $\sqrt{x+2} = 10$
12. $2\sqrt[3]{3x-4} = 6$
13. $(x+3)^{\frac{2}{3}} - 3 = 1$
14. $\sqrt{x+10} = x + 1$
15. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$, where V is the volume and r is the radius of the sphere. Find the radius of a sphere with a volume 4 ft^3.

5-05

Let $f(x) = x + 2$, and $g(x) = x^2$. Perform the indicated operation.

16. $f(x) - g(x)$
17. $f(x) \cdot g(x)$

5-06

18. $f(g(x))$
19. $g(f(x))$

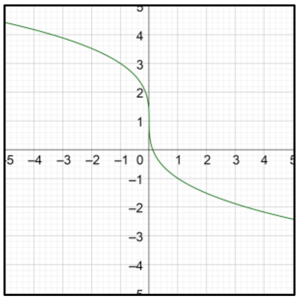
5-07

Find the inverse of the function.

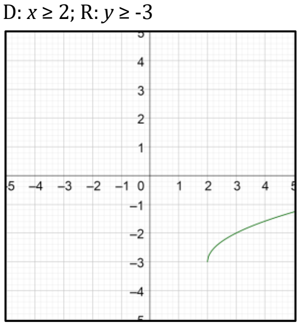
20. $f(x) = 64x^3$
21. $g(x) = x^{10} - 2, x \leq 0$
22. $h(x) = 2(x)^4, x \geq 0$

Answers

1. 3.50
2. 125
3. $-1, 3$
4. q^3
5. $\frac{x^4}{3}$
6. $5\sqrt[3]{3}$
7. $2xy^2\sqrt[5]{2x^3}$
8. D: All real; R: All real



9. D: $x \geq 2$; R: $y \geq -3$



10. Vertical stretch by factor of 2 and translate 3 left
11. 98
12. $\frac{31}{3}$
13. 5
14. $\frac{-1+\sqrt{37}}{2}$ ($\frac{-1-\sqrt{37}}{2}$ is extraneous)
15. 0.98 ft
16. $-x^2 + x + 2$
17. $x^3 + 2x^2$
18. $x^2 + 2$
19. $x^2 + 4x + 4$
20. $y = \frac{\sqrt[3]{x}}{4}$
21. $y = -\sqrt[10]{x+2}$
22. $y = \sqrt[4]{\frac{x}{2}}$