## Algebra 2

## 5-01 nth Roots and Rational Exponents

## Root

- If $a^{2}=b$, then $a$ is a $\qquad$ $\left(2^{\text {nd }}\right)$ root of $b$.
- If $a^{n}=b$, then $a$ is the $\qquad$ root of $b$.


## Parts of a radical



## Rational Exponents

$$
\begin{gathered}
b^{1 / n}=\sqrt[n]{b} \\
b^{m / n}=\sqrt[n]{b^{m}}=(\sqrt[n]{b})^{m}
\end{gathered}
$$

Evaluate
$36^{1 / 2}$
$\left(\frac{1}{8}\right)^{-\frac{1}{3}}$
$27^{\frac{4}{3}}$

Find roots with a calculator

- The $\sqrt{x}$ or $\sqrt{ }$ key is for $\qquad$ roots (either radicand then key or key then radicand depending on calculator)
- The $\sqrt[x]{y}$ or $\sqrt[y]{x}$ or $\sqrt[x]{ }$ is for $\qquad$ root (index $\rightarrow$ key $\rightarrow$ radicand OR radicand $\rightarrow$ key $\rightarrow$ index)
Try it with $\sqrt[4]{100}$


## Steps to solve an equation with an exponent

1. $\qquad$ the exponent term
2. Take the $\qquad$ of both sides where the index is the $\qquad$

- If the index is $\qquad$ , put $\qquad$

3. $\qquad$
4. $\qquad$ your answers!!!

Solve. Round to two decimal places, if necessary.
$5 x^{3}=320$

$$
(x+3)^{4}=24
$$

## Algebra 2

5-02A Properties of Rational Exponents and Simplifying Radicals

## Properties of Rational Exponents

- $x^{m} \cdot x^{n}=x^{m+n}$
- $\quad(x y)^{m}=x^{m} y^{m}$
- $\left(x^{m}\right)^{n}=x^{m n}$
- $\frac{x^{m}}{x^{n}}=x^{m-n}$
- $\left(\frac{x}{y}\right)^{m}=\frac{x^{m}}{y^{m}}$
- $x^{-m}=\frac{1}{x^{m}}$
$6^{\frac{1}{2}} \cdot 6^{\frac{1}{3}}$

$$
\left(27^{\frac{1}{3}} \cdot 6^{\frac{1}{4}}\right)^{2}
$$

$\left(4^{3} \cdot w^{3}\right)^{-\frac{1}{3}}$
$\frac{t}{t^{\frac{3}{4}}}$

## Simplifying Radicals

Remove any $\qquad$ roots
Rationalize $\qquad$
$\sqrt[4]{64} \sqrt[3]{625 x^{5}}$

$242 \# 1,3,5,7,9,19,21,23,25,27,29,45,47,49,95=15$

## Algebra 2

5-02B Operations with Radicals

## Using Properties of Radicals

Product Property $\rightarrow \sqrt[n]{a \cdot b}=\sqrt[n]{a} \cdot \sqrt[n]{b}$
Quotient Property $\rightarrow \sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
$\sqrt[3]{25} \cdot \sqrt[3]{5} \quad \sqrt[3]{\sqrt[3]{4 x}}$

Adding and Subtracting Roots and Radicals

1. Simplify the $\qquad$
2. like terms
$5\left(4^{\frac{3}{4}}\right)-3\left(4^{\frac{3}{4}}\right)$

$$
\sqrt[3]{81}-\sqrt[3]{3}
$$

$2 \sqrt[4]{6 x^{5}}+x \sqrt[4]{6 x}$

## Algebra 2

## 5-03 Graphing Radical Equations

$y=\sqrt{x}$
Domain: $\qquad$
Range: $\qquad$
$y=\sqrt[3]{x}$
Domain: $\qquad$


- Where
- $a$ $\qquad$ by factor of $a$
- If $b$ is -, $\qquad$ over $\qquad$
- b $\qquad$ by factor of $\frac{1}{b}$
- h $\qquad$
- If $a$ is -, $\qquad$ over $\qquad$
- $k$ $\qquad$
- Graph by making a $\qquad$ .

Describe the transformation of $f$ represented by $g$. Then graph each function.
$f(x)=\sqrt{x} ; g(x)=\sqrt{x+2}-3$

$f(x)=\sqrt[3]{x} ; g(x)=-\sqrt[3]{2 x}$


The function $E(d)=0.25 \sqrt{d}$ approximates the number of seconds it takes a dropped object to fall $d$ feet on Earth. The function $J(d)=0.63 \cdot E(d)$ approximates the number of seconds it takes a dropped object to fall $d$ feet on Jupiter. How long does it take a dropped object to fall 81 feet on Jupiter?
$\qquad$

Let the graph of $g$ be a horizontal stretch by a factor of 3 , followed by a translation 6 units right of the graph of $f(x)=\sqrt[3]{x}$. Write a rule for $g$.

## Graphing horizontal parabolas and circles

1. $\qquad$ the equation for $y$.
2. Create a $\qquad$ .
3. $\qquad$ the points and $\qquad$ graph.
Graph $-\frac{1}{5} y^{2}=x$. Identify the vertex and the direction that the parabola opens.


Graph $x^{2}+y^{2}=49$. Identify the radius and the intercepts.


## Algebra 2

5-04 Solving Radical Equations and Inequalities

## Radical Equation

Equation containing a $\qquad$

## Steps to Solve a Radical Equation

1. $\qquad$ the radical
2. $\qquad$ both sides to whatever the $\qquad$ is (or the reciprocal of the exponent)
3. 
4. your answers!!!
$5-\sqrt[4]{x}=0 \quad 3 x^{\frac{4}{3}}=243$
$\sqrt{2 x+8}-4=6 \quad \sqrt{4 x+28}-3 \sqrt{2 x}=0$
$x+2=\sqrt{2 x+28}$

## Algebra 2

## 5-05 Performing Function Operations

## Ways to combine functions

- Addition:
$(f+g)(x)=f(x)+g(x)$
- Subtraction:
$(f-g)(x)=f(x)-g(x)$
- Multiplication:
$(f \cdot g)(x)=f(x) \cdot g(x)$
- Division:
$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$
Given $f(x)=5 \sqrt{x}$ and $g(x)=-8 \sqrt{x}$ find
$(f+g)(x)$

$$
(f-g)(x)
$$

$(f \cdot g)(x)$ $\left(\frac{f}{g}\right)(x)$

Let $f(x)=2 x^{3}+4 x^{2}-8 x+4$ and $g(x)=3 x^{3}-5 x^{2}+6 x-9$. Find $(f-g)(x)$ and state the domain. Then evaluate $(f-g)(-1)$.

Let $f(x)=x^{3}$ and $g(x)=\sqrt{x}$. Find $(f g)(x)$ and state the domain. Then evaluate $(f g)(4)$.

From 2010 to 2020, the populations (in thousands) of City M and City N can be modeled by $M(t)=3.3 t^{3}+12.1 t^{2}-0.65 t+$ 15.8 and $N(t)=2.5 t^{3}+7.8 t^{2}+0.41 t+11.9$, where $t$ is the number of years since 2010 . Find $(M-N)(t)$ and explain what it represents.
$265 \# 1,3,5,7,9,15,17,21,23,25,27,29,35,37,39=15$

## Algebra 2

## 5-06 Composition of Functions

## Composition

- Put one function $\qquad$ the other. (Like $\qquad$ _)
- Written $\qquad$
- Said " $g$ of $f$ of $x$ "
- Means that the $\qquad$ (range) of $f$ is the $\qquad$ (domain) of $g$. Work from the inside out. Do $f(x)$ first then $g(x)$.
- $f(x)$ gets $\qquad$ into $g(x)$
Let $f(x)=\sqrt{3 x-5}$ and $g(x)=x^{2}+1$. Find the indicated value.

a. $g(f(2))$
b. $f(g(3))$
c. $g(g(-3))$

Let $f(x)=3 x^{-1}$ and $g(x)=4 x-5$. Perform the indicated operation and state the domain.
a. $f(g(x))$
b. $g(f(x))$
c. $f(f(x))$

Algebra 2 5-06
Name:
The function $C(x)=8.74 x$ represents the cost (in dollars) of producing $x$ shirts. The number of shirts produced in $t$ hours is represented by $x(t)=84 t$. (a) Find $C(x(t))$. (b) Evaluate $C(x(40))$ and explain what it represents.
$271 \# 1,5,9,13,17,21,25,31,33,37,43,45,47,49,51=15$

## Algebra 2

## 5-07 Inverse of a Function

## Properties of Inverses

- $x$ and $y$ values are $\qquad$
- Graph is $\qquad$ over the line $\qquad$
- You can use the Horizontal Line test to determine if the $\qquad$ of a function is also a function.
- If a horizontal line can touch a graph $\qquad$ then the inverse is $\qquad$ a function.


## Definition of inverses

- Two functions are inverses if and only if $\qquad$ and $\qquad$
Verify that $f(x)=6-2 x$ and $g(x)=\frac{6-x}{2}$ are inverses.


## Finding inverses

- Inverses switch the $x$ and $y$ $\qquad$

1. $\qquad$ $x$ and $y$ and $\qquad$ for $y$.
Find the inverse
$y=2 x+7$

$$
f(x)=x^{4}+2, x \leq 0
$$

The power (in watts) of a lightbulb that has a resistance of 240 ohms is represented by $f(x)=240 x^{2}$, where $x$ is the electric current of a lightbulb in amperes. Find and interpret $f^{-1}(60)$.
$\qquad$

## Algebra 2

## 5-Review

Take this test as you would take a test in class. When you are finished, check your work against the answers. 5-01

1. Evaluate $\sqrt[4]{150}$ using a calculator. Round the result to two decimal places if appropriate.
2. Evaluate $25^{\frac{3}{2}}$ using a calculator. Round the result to two decimal places if appropriate.
3. Solve $128=2(x-1)^{6}$

## 5-02

Simplify the expression. Assume all variables are positive.
4. $q^{\frac{7}{3}} \cdot q^{\frac{2}{3}}$
5. $\frac{x^{10}}{3 x^{6}}$
6. $\sqrt[3]{81}+\sqrt[3]{24}$
7. $\sqrt[5]{64 x^{8} y^{10}}$

5-03
Graph the function. Then state the domain and range.
8. $y=-2 \sqrt[3]{x}+1$
9. $y=\sqrt{x-2}-3$
10. Describe the transformations to get $g(x)=2 \sqrt[3]{x+3}$ from $f(x)=\sqrt[3]{x}$.

## 5-04

Solve the equation.
11. $\sqrt{x+2}=10$
12. $2 \sqrt[3]{3 x-4}=6$
13. $(x+3)^{\frac{2}{3}}-3=1$
14. $\sqrt{x+10}=x+1$
15. The volume of a sphere is given by $V=\frac{4}{3} \pi r^{3}$, where $V$ is the volume and $r$ is the radius of the sphere. Find the radius of a sphere with a volume $4 \mathrm{ft}^{3}$.

## 5-05

Let $f(x)=x+2$, and $g(x)=x^{2}$. Perform the indicated operation.
16. $f(x)-g(x)$
17. $f(x) \cdot g(x)$

5-06
18. $f(g(x))$
19. $g(f(x))$

5-07
Find the inverse of the function.
20. $f(x)=64 x^{3}$
22. $h(x)=2(x)^{4}, x \geq 0$
21. $g(x)=x^{10}-2, x \leq 0$

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$\qquad$
Answers

1. 3.50
2. 125
3. $-1,3$
4. $q^{3}$
5. $\frac{x^{4}}{3}$
6. $5 \sqrt[3]{3}$
7. $2 x y^{25} \sqrt{2 x^{3}}$
8. D: All real; R: All real

9. D: $x \geq 2 ; \mathrm{R}: y \geq-3$

10. Vertical stretch by factor of 2 and translate 3 left
11. 98
12. $\frac{31}{3}$
13. 5
14. $\frac{-1+\sqrt{37}}{2}\left(\frac{-1-\sqrt{37}}{2}\right.$ is extraneous)
15. 0.98 ft
16. $-x^{2}+x+2$
17. $x^{3}+2 x^{2}$
18. $x^{2}+2$
19. $x^{2}+4 x+4$
20. $y=\frac{\sqrt[3]{x}}{4}$
21. $y=-\sqrt[10]{x+2}$
22. $y=\sqrt[4]{\frac{x}{2}}$
